Introductory section – the answers can be found in the text of the notes and slides. The purpose is to warm up the concepts and terms.

1. Steam flows into the low pressure turbine of a steam power plant at 250 m/s at 20 bar and 400°C. The speed of sound in steam at these conditions is 623 m/s. Calculate the Mach number of this flow and state whether compressibility is likely to affect the flow. Derive the stagnation enthalpy for this condition if stagnation is achieved isentropically.
2. Calculate the Mach number, and stagnation conditions, for air at 315 K and 180 kPa flowing at 250 m/s. What mass rate is possible through a nozzle of 0.1 m2 aperture? Assume no vena-contractor.
3. Calculate the static temperature for a flow at Mach 2 which experiences a shock, when the static temperature before the shock 250 K and static pressure 0.4 bar.
4. Describe why weak compressibility effects can be considered as isentropic, but flow through shock waves is adiabatic but not isentropic.
5. The Mach number is *M*=*c*/*a*, where *c* is the speed of the fluid and *a* is the speed of sound. In this case it is 250/623 = 0.40. The point at which compressibility starts to become significant is when *M* >0.3, and therefore this is compressible, weakly. For isentropic stagnation, the stagnation enthalpy will be:

$$h\_{0}=h+\frac{1}{2}c^{2}$$

The enthalpy of steam at 20 bar and 400°C is 3246 *kJ/kg*. Therefore, remembering that the kinetic term will have units of *J/kg*:

$$h\_{0}=3,246,000+\frac{1}{2}250^{2}=3277 kJ/kg$$

1. Firstly obtain the speed of sound in air at 315 *K* using *a* = ÖgR*T = Ö1.4\*287\*315 = 356 m/s.*

M = 250/356 = 0.7, which is just transonic.

For the mass flow, use the formula in the notes, and need the stagnation temperature, *T0*:

$$\frac{T\_{0}}{T}=1+\frac{γ-1}{2}M^{2}$$

$$T\_{0}=315\left(1+\frac{1.4-1}{2}0.7^{2}\right)=346 K$$

$$\frac{\dot{m}\sqrt{c\_{p}T\_{0}}}{A\_{n}p\_{0}}=\frac{γ}{\sqrt{γ-1}}M\left(1+\frac{γ-1}{2}M^{2}\right)^{\frac{1}{2}\left(\frac{γ+1}{γ-1}\right)}$$

$$\dot{m}=\frac{A\_{n}p\_{0}}{\sqrt{c\_{p}T\_{0}}}\frac{γ}{\sqrt{γ-1}}M\left(1+\frac{γ-1}{2}M^{2}\right)^{\frac{1}{2}\left(\frac{γ+1}{γ-1}\right)}$$

$$\dot{m}=\frac{0.1×180,000}{\sqrt{1005×346}}\frac{1.4}{\sqrt{0.4}}0.7\left(1+\frac{1.4-1}{2}0.7^{2}\right)^{\frac{1}{2}\left(\frac{1.4+1}{1.4-1}\right)}=62 kg/s$$

1. This question relies on using the formulae from thermofluids 2 for normal shocks. You can use the spreadsheet with the calculations in as a check, but for this example you should convince yourself that you can calculate it as follows:

$$\frac{p\_{2}}{p\_{1}}=\frac{1}{γ+1}\left[2γMa\_{1}^{2}-\left(γ-1\right)\right]$$

$$\frac{T\_{2}}{T\_{1}}=\left[2+\left(γ-1\right)Ma\_{1}^{2}\right]\frac{2γMa\_{1}^{2}-\left(γ-1\right)}{\left(γ+1\right)^{2}Ma\_{1}^{2}}$$

Use g=1.4 as a typical value for air

$$\frac{T\_{2}}{250}=\left[2+\left(1.4-1\right)2^{2}\right]\frac{2×1.4×2^{2}-\left(1.4-1\right)}{\left(1.4+1\right)^{2}2^{2}}=3.6×\frac{10.8}{23.04}=1.688\rightarrow T\_{2}=422.0 K$$

The question doesn’t ask for the pressure, but because it gives the static pressure prior to the shock, 40 kPa, we can also have the static pressure after the shock:

$$\frac{p\_{2}}{40}=\frac{1}{1.4+1}\left[2×1.4×2^{2}-\left(1.4-1\right)\right]=\frac{10.8}{2.4}=4.5\rightarrow p\_{2}=180 kPa$$

Confirm this by reference to the shock tables, for M1 = 2:

$$\frac{T\_{2}}{T\_{1}}=1.6875, \frac{p\_{2}}{p\_{1}}=4.5$$

Therefore the answers are correct.

1. For weakly compressible flows, up to Mach 1, prior to a strong shock wave, the flow is isentropic because there is no heat transfer in the flow and there are no strong irreversible effects due to viscosity. Where there is a shock, the friction effects caused across the shock, which lead to a rise in static pressure, are due to friction, so although adiabatic, they are not reversible due to friction.

Compressible flow questions - intended to highlight key areas for knowledge and to demonstrate key estimation and calculation techniques.

1. Sketch the velocity vectors that result from an oblique shock and state what happens to the vectors parallel and perpendicular to the shock wave.
2. Flow approaches a plane wall at an angle of 15° to the incoming flow, which has a Mach number of 3. Find the angle of the weak and strong shocks and hence the resulting Mach number. [Ans: 2.6**6**, 0.**88**] bold indicates correction from notes original.
3. Describe the characteristics of Prandtl-Meyer expansion fans and why they occur in that way rather than an abrupt shock.
4. A flow of air at Mach 3 expands around a corner of angle 30°. Use the Prandtl-Meyer chart to determine the exit Mach number. [Ans: Mach ~5.3]
5. A pipe has a wall friction factor of 0.0015 and a diameter of 0.3 m, **with initial Mach number of 0.2 in air**. Use the chart for pipe flow with friction to find the length required to result in choked flow. [Ans: 525 m]
6. Flow of air in a pipe has stagnation temperature of 400 K and specific heat capacity at constant pressure of 1.005 kJ/kgK, and flows at Mach 0.4. Use the chart for critical specific heat addition to find the specific heat necessary to cause choking in the flow. [Ans: 362 kJ/kg]
7. Obtain the sketch from section 9b



resolving the approach velocity c1 into components parallel and perpendicular to the oblique shock, the perpendicular component only is affected by the shock. The parallel component remains unchanged. The resultant velocity of the downstream components parallel and perpendicular to the shock result in a flow parallel to the plane angle which caused the shock.

1. Make use of the chart in section 9b.



From the shock wave angle, q, corresponding to the Mach number 3 and the deflection angle 15°, as indicated by the dashed lines, q = 32° for the weak shock and 84° for the strong shock. Taking each angle in turn produces the downstream results via trigonometry.

final point is M2 = M2N/sin (b-d)

to get there:

M1N = M1sin b

Which for M1 = 3 and for 32° is 1.590 for the weak shock and for 84° is 2.984 for the strong shock.

Then use the normal shock chart for M2N in each case:

M2N,weak = 0.6715

M2N,strong = 0.5362

Using M2 = M2N/sin (b-d) in each case:

M2,weak = 0.6715/sin(32-15) = 2.300

M2,strong = 0.5362/sin(84-15) = 0.574

1. An expansive abrupt shock is not possible, because it would theoretically produce a reduction in entropy, which is not possible. Instead, when an expansion occurs a series of infinitesimal strength Mach waves occurs until the flow direction is accordingly adjusted. These are called Prandtl-Meyer fans.
2. Use the chart in section 9c



Starting at Mach 3, we see that an initial M=1 flow would have turned 50° to make it reach M=3. Now we turn a further 30°, so up to total 80°, and the corresponding Mach number is M=5.5.

1. For a pipe with diameter 0.3 m and a wall roughness of 0.0015 m with starting Mach number 0.2, use the formula for frictional flow in a pipe or use the chart:

$$4C\_{f}\left(\frac{L^{\*}}{D}\right)=\frac{1}{γ}\left[\frac{1-M^{2}}{M^{2}}\right]+\frac{γ+1}{2γ}ln\frac{\left(γ+1\right)M^{2}}{2+\left(γ-1\right)M^{2}}$$

$$4×0.0015\left(\frac{L^{\*}}{0.3}\right)=10.5\rightarrow L^{\*}=525 m$$



1. For stagnation temperature of 400 K, and starting Mach number is 0.4 in air, use the chart or formula for heat addition to find how much heat flux is necessary in a pipe to produce choked flow.

$$\frac{q^{\*}}{C\_{p}T\_{0}}=\frac{T\_{0}^{\*}}{T^{\*}}\frac{T}{T\_{0}}\frac{T^{\*}}{T}-1=\frac{1}{1+\frac{γ-1}{2}M^{2}}\left[\frac{1+γM^{2}}{M\left(1+γ\right)}\right]^{2}\left(1+\frac{γ-1}{2}\right)=0.89$$

Therefore:

$$q^{\*}=0.89×C\_{p}T\_{0}=0.89×1.005×400=358 kJ/kg$$

The mass flux can then be related to the heat transfer coefficient involved on the pipe wall to see if this critical heat gain is achieved.